The Idealization Problem for Bayesianism

Lukáš Bielik
Comenius University in Bratislava

Science, Philosophy of Science and Scientific Methodology II Conference (on-line), The University of Montenegro

November 20, 2025

Motivation

- Bayesianism
  - a successful theory of confirmation with many applications: evidence diversity, robustness analysis, explanatory power …
- Idealization
  - a deliberately accepted false statement (assumption) fulfilling a theoretical function
- Problem (Shaffer 2001; 2012)
  - How could an idealized theory / model be confirmed in Bayesian Confirmation Theory (BCT)?

Plan

1. Representing theoretical idealizations
2. Bayesianism
3. The problem of idealization
4. Bayesian solutions
5. Extension: The Best Model Presupposition

1. Representing theoretical idealizations

- Two ways
  - Explicit (Shaffer 2001; 2012)
  - Implicit (Sprenger and Hartmann 2019; Vassend 2019; Sprenger 2020)
- Explicit representation
  - Counterfactuals: I > T

    If I were the case, then T would hold.

    If a real system's frictional forces were zero, the dynamics of an object's free fall would follow this equation: $d^2y/dt^2 = -g$
- Implicit representation
  - 'T', 'H' or 'θ' represent a (complex) theoretical statement with idealized assumptions.

2. Bayesianism

- BCT as the most developed theory of confirmation
- Centra tenet – Bayes' Theorem
  - $P(H|E) = P(H) \cdot P(E|H) / P(E)$
    - P(H) – prior probability
    - P(E|H) – likelihood
    - P(E) – expectedness of hypothesis
    - P(H|E) – posterior probability
  - H is confirmed by E iff $P(H|E) > P(H)$

3. The Problem of Idealization

- Shaffer's formulation:
  - 'I > T' for 'H' in BT:
  - $P(I > T|E) = P(I > T) \cdot P(E|\ I > T)\ /\ P(E)$
  - How to assign probabilities to (I > T)?
    - $P(I > T) = P(T|I) = P(T \wedge I)\ /\ P(I)$, provided $P(I) > 0$. ×
    - Lewis's (1976) imaging procedure ×
    - AGM/Levi approach ×

3. The Problem of Idealization

- Vassend's (2019) formulation
  - The interpretation problem:

    It is almost invariably the case in regression problems that the hypotheses under consideration will be restricted to very simple functional relationships, such as the set of lines, parabolas, exponentials, and so on. Most functional relationships in the world cannot realistically be expected to belong to one of these sets of simple functional relationships, and indeed the choice of functional class is usually justified on the basis of highly idealized scientific assumptions, if it is justified at all. … If scientists were to assign a probability of 0 to all functional relationships they know to be false, they would in effect rule out all of their hypotheses from the get-go. (Vassend 2019, 700).

3. The Problem of Idealization

- Sprenger and Hartmann (2019)'s formulation
  - A trilemma:

1) In Bayesian inference, P(H) denotes an agent's degree of belief that H is true.
2) H is part of a general statistical model **M** with the partition of hypothesis **H** = {H1, …, Hn}.
3) Many such models **M** are strong idealizations of reality, hence (very likely) false.

1), 2) and 3) together inconsistent

4. Bayesian solutions

- Vassend's proposal
  - One option: A different algebra of propositions of the form:

    <θ_i is the best hypothesis>
    - Problem: likelihoods P(E | <θ_i is the best hypothesis>) have no reasonable predictive values
  - Preferred option: a verisimilitude interpretation of $P(\theta)$
    - $P^v(\theta)$ = the probability that θ maximizes *v*.

4. Bayesian solutions

- Evaluation of Vassend's proposal
  - Let us refer to <θ_i is the best hypothesis> by 'γ_i'
  - The hypotheses space ***H*** = $\{\gamma_1|\theta_1 \ldots, \gamma_n|\theta_n\}$
  - Prior probabilities of the form $P(\gamma_i|\theta_i)$ are defined (for any i).
  - Moreover: The likelihoods of the form $Pr(E|\ \theta_i \wedge \gamma_i)$ are defined even though hypothesis θ_i screens off γ_i from E, and hence, we can assign likelihood values directly to $Pr(E|\theta_i)$.
    - The relation of screening-off: θ_i screens off γ_i from E iff $P(E|\ \theta_i \wedge \gamma_i) = P(E|\theta_i)$ and $P(E|\ \neg\theta_i \wedge \gamma_i) = P(E|\neg\theta_i)$

4. Bayesian solutions

- Drawbacks of the verisimilitude solution
  - Different dimensions of models' evaluation (simplicity AND accuracy)
  - A caveat wrt $P^v(\theta)$: composite disjunctive hypotheses of the form $\theta_i \vee \theta_j$ have greater probability of maximizing *v* than simpler ones (like θ_i)
  - The greater probability – the smaller informativeness
  - A neglect of other theoretical / model-relevant virtues

4. Bayesian solutions

- Sprenger and Hartmann's solution: Suppositional Analysis
  - Motivation: to justify $P(E|H) = \rho_H(E)$ (EQ)
  - Let ω_H represents any possible world where H is true. Then: $P_{\omega H}$ (E) expresses the probability that we will observe E upon the supposition of being in a world ω governed by H.
  - To appeal to some chance–credence coordination principle (e.q. the Principal Principle) to motivate the equality $Pr_{\omega H}(E) = \rho_H(E)$.
  - Second step: generalize this strategy and relativize all terms in Bayes' Theorem to some general statistical model ***M***.

4. Bayesian solutions

- Sprenger and Hartmann's solution …
  - A model-relative version of Bayes' Theorem:

    $P_M(H|E) = P(H)_M \cdot P_M(E|H) / P_M(E)$ (BT^M)
  - We take $P_M(H)$ to be a conditional counterfactual degree of belief "that we would have in H, if we supposed that the target system is fully and correctly described by one of the hypothesis in ***M***"(Sprenger and Hartmann 2019, 312).
  - The similar interpretation applies to likelihoods $P_M(E|H)$, the expectedness of evidence ($P_M(E)$) and the posterior probability $P_M(H|E)$.

4. Bayesian solutions

- Evaluation of Sprenger and Hartmann’s solution
  - What exactly does it mean to say that “the target system is fully and correctly described by one of model’s hypothesis”?
    - Completeness and accuracy as the only relevant aspects to the evaluation of model-based statistical reasoning?
    - The problems similar to those of Vassend’s proposal…

5. Extension: The best model presupposition

- Preliminary assumptions
  - Scientific modeling is motivated by a set of research goals.
  - Research goals are context-sensitive.
  - The importance of general theoretical (or model-relative) virtues (e.g. accuracy, simplicity, explanatory scope, informativeness, ...)
  - A trade-off of these model virtues enable us to prefer one model in some research context and another model in a different context.
  - A set of research goals influences the preference and a particular weight of specific model virtues which, in turn, help us to choose one model as more adequate than another one in a given context.

5. Extension: The best model presupposition

- Comparative adequacy of models
  - For any given research context C and some context-sensitive trade-off W between the modeling virtues V we say that model M_i is more adequate than model M_j if and only if M_i exemplifies W in C to a greater extent than M_j.
- Applying an adequate model in inference
  - To interpret '$P_M(H)$' as the probability that H is the most empirically adequate on the supposition that M is the most adequate model.
  - Similarly with '$P_M(E|H)$', '$P_M(E)$', '$P_M(H|E)$'

5. Extension: The best model presupposition

- Conclusion
  - If Bayesianism has to deal with idealized models and theories, it should incorporate not only those aspects of modeling that maximize truth and accuracy, but also those features of modeling that speak for the overall adequacy of models we use in scientific inference.

THANK YOU!

The work on this paper was supported by project VEGA-1/0557/23 and project APVV-21-04-05.

References

- Cartwright, N. (1983): *How the Laws of Physics Lie*. New York: Oxford University Press.
- Frigg, R. & Hartmann, R. (2025): Models in Science. In: Zalta, E. N. & Nodelman, U. (eds.): *The Stanford Encyclopedia of Philosophy* (Summer 2025 Edition), available at:

  URL = <https://plato.stanford.edu/archives/sum2025/entries/models-science/>.
- Jones, M. R. (2005): Idealization and Abstraction: A Framework. In: Jones, M. R. and Cartwright, N. (eds.): *Idealization XII. Correcting the Model*. Amsterdam: Rodopi, pp. 173–217.
- Nowak, L. (1980): *The Structure of Idealization: Towards a Systematic Interpretation of the Marxian Idea of Science*. Dordrecht: Springer.
- Potochnik, A. (2017): *Idealization and the Aims of Science*. Chicago, IL: University of Chicago Press.
- Shaffer, M. J. (2012): *Counterfactuals and Scientific Realism*. New York: Palgrave Macmillan.

References ...

- Shaffer, M. J. (2001): Bayesian Confirmation of Theories That Incorporate Idealizations. *Philosophy of Science* 68, No. 1, pp. 36–52.
- Sprenger, J. (2020): Conditional Degree of Belief and Bayesian Inference. *Philosophy of Science*, 87, pp. 319–335.
- Sprenger, J. and Hartmann, S. (2019): *Bayesian Philosophy of Science*. Oxford: Oxford University Press.
- Vassend, O. B. (2019): New Semantics for Bayesian Inference: The Interpretive Problem and Its Solutions. *Philosophy of Science*, 86, pp. 696–718.
- Weisberg, M. (2013): *Simulation and Similarity: Using Models to Understand the World*. Oxford: Oxford University Press.