

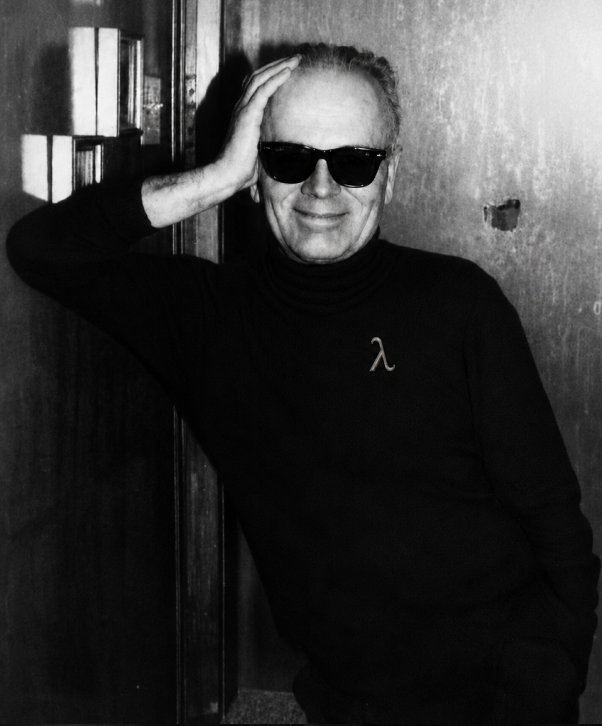
Analytically True Conditionals in Transparent Intensional Logic

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What?

- Can we provide a uniform account of analytically true conditionals?
- To do so, we need to cover also analytically true counterpossibles, assuming that some of them are non-trivially analytically true (this calls for a sufficiently robust framework)
- We also need an account that can capture analytic truth and essential relations between meanings
- TIL satisfies these requirements, so we chose this framework

Why?

- Across various sorts of conditionals (indicative, counterfactual, even counterpossible or deontic), some of them seem non-trivially analytically true:
 - If Tim is a kitten, Tim is a cat.
 - If Tina had been a bachelor, Tina would have been a man.
 - If Tim had been an adult kitten, Tim would have been a kitten.
 - If A and $\neg A$, then A .
 - If it's prohibited to steal groceries, it is prohibited to steal cheese.
- The debate on conditionals has focused a lot on empirically true conditionals and a semantically interesting category of (non-trivially) analytically true conditionals of various sorts have not been covered yet, not even in TIL, a framework well suited for this task



How?

- Ingredient 0: review of the debate on conditionals
- Ingredient 1: analytic truth (Materna, Duží)
- Ingredient 2: essential relationships between concepts: hyperrequisites (Duží, Jespersen, Glavaničová)
- Ingredient 3: from hyperrequisites to hyperpropositions
- Outcome: a TIL analysis of analytically true conditionals

Ingredient 1: analytic truth

- LANL Principle (Materna 2007 quoting Tichý 1966/2004): “We assume, of course, a normal linguistic situation, in which communication proceeds between two people, both of whom understand the language. Logical semantics does not deal with other linguistic situations.”
- analytic truth (Materna 2007):
 - A sentence is analytically true iff its meaning constructs T (mathematical sentence) or the proposition $TRUE$, where T is a truth-value (truth) and $TRUE$ is a constant function that always takes T as a value (in all possible worlds and times).

Materna, P. (2007). Once more on analytic vs. synthetic. *Logic and Logical Philosophy*, 16(1), 3–43.

Duží, M. (2010). The paradox of inference and the non-triviality of analytic information. *Journal of Philosophical Logic*, 39(5), 473–510.

Ingredient 2: essential relationships between concepts: hyperrequisites

- Speaking about “impossible individuals” without impossible individuals-in-extension
- Impossible individuals as necessarily empty individual concepts
- Explosion: every property is a requisite of an impossible role, so requisites and individuals-in-intension are not enough:

$$[{}^0Req p q] = [{}^0\forall \lambda w [{}^0\forall \lambda t [[{}^0Occ_{wt} q] \supset [{}^0True_{wt} \lambda w \lambda t [p_{wt} q_{wt}]]]]].$$

Duží, M., Jespersen, B., & Glavaničová, D. (2021). Impossible individuals as necessarily empty individual concepts. *Logic in High Definition: Trends in Logical Semantics*, 177–202.

Ingredient 2: essential relationships between concepts: hyperrequisites

- *Primary hyperrequisite of a hyperoffice*: Let $*Off/*_n \rightarrow l_{\tau\omega}$; hyperrequisite $*Req/*_n \rightarrow (o(o\iota)_{\tau\omega} l_{\tau\omega})$. Then the *primary hyperrequisites* $*Req$ of the hyperoffice $*Off$ are those property-producing constructions that are provably derivable from $*Off$ without applying *ex falso quodlibet*.

Duží, M., Jespersen, B., & Glavaničová, D. (2021). Impossible individuals as necessarily empty individual concepts. *Logic in High Definition: Trends in Logical Semantics*, 177–202.

Ingredient 2: essential relationships between concepts: hyperrequisites

- This definition is to be applied in an iterated manner: one infers further constructions from the construction in the previous step. If the construction is in the form 0F (trivialisation of property F), then one uses the method of *refinement*, which consists in replacing an atomic construction by an equivalent molecular construction that defines the property F . In the second step one infers the set $*Req_1$, which contains constructions that are provably derivable from 'refined' elements of $*Req_0$.

Duží, M., Jespersen, B., & Glavaničová, D. (2021). Impossible individuals as necessarily empty individual concepts. *Logic in High Definition: Trends in Logical Semantics*, 177–202.

Ingredient 3: from hyperrequisites to hyperpropositions

- *Primary consequence of a hyperproposition:* Let $*Prop/*_n \rightarrow o_{\tau\omega}$; consequence $*Cons/*_n \rightarrow (o o_{\tau\omega} \iota_{\tau\omega})$. Then the primary consequences $*Cons$ of a hyperproposition $*Prop$ are those propositional constructions that are provably derivable from $*Prop$ without employing *ex falso quodlibet*.
- *By iteration one infers further construction from constructions in the previous step.* If a construction contains 0X (trivialisation of X), then one uses the method of *refinement*, which consists in replacing the atomic construction by an equivalent molecular construction that defines X . Subsequently, one infers the set $*Cons_1$, which contains constructions that are provably derivable from “refined” elements of $*Cons_0$.

Outcome: a TIL analysis of analytically true conditionals

- If one finds out in a finite number of steps n and m that consequences $*Cons_m$ of a hyperproposition in the consequent are contained in the consequences $*Cons_n$ of the hyperproposition in the antecedent, the conditional is analytically true.
 - (1) If Tim is a kitten, Tim is a cat.
 - (2) If Tina had been a bachelor, Tina would have been a man.
 - (3) If Tim had been an adult kitten, Tim would have been a kitten.
 - (4) If A and $\neg A$, then A .
 - (5) If it's obligatory that Tilman cleans up and cooks, it is obligatory that Tilman cooks.

(1): a TIL analysis of analytically true conditionals

- $\lambda w \lambda t [[^0Kitten_{wt} ^0Tim] \rightarrow [^0Cat_{wt} ^0Tim]]$
- The $*Cons_0(A)$ of the hyperproposition in the antecedent are as follows: $*Cons_0(A)$ antecedent: $\{[^0Kitten_{wt} ^0Tim]\}$
- And the $*Cons_0(C)$ of the hyperproposition in the consequent are as follows: $*Cons_0(C)$ consequent: $\{[^0Cat_{wt} ^0Tim]\}$
- Those in the antecedent contain a molecular construction 0Kitten , which we can refine as a cat that is not adult ($[[\neg ^0Adult_{wt} ^0Tim] \wedge [^0Cat_{wt} ^0Tim]]$) and remove the conjunction.

(1): a TIL analysis of analytically true conditionals

- The consequences $*Cons_1(C)$ of the hyperproposition in the consequent are then as follows: $*Cons_1(A)$ antecedent (refined Kitten): $\{[{}^0Kitten_{wt} {}^0Tim], [[\neg {}^0Adult_{wt} {}^0Tim] \wedge [{}^0Cat_{wt} {}^0Tim]], [\neg {}^0Adult_{wt} {}^0Tim], [{}^0Cat_{wt} {}^0Tim]\}$
- Given that $[{}^0Cat_{wt} {}^0Tim]$ is contained in $*Cons_1(A)$, we get:
 $*Cons_0(C) \subseteq *Cons_1(A)$
- The consequences $*Cons_0(C)$ of a hyperproposition in the consequent are contained in the consequences $*Cons_1(A)$ of the hyperproposition in the antecedent, so the conditional is analytically true.

(2*): a TIL analysis of analytically true conditionals

- similarly:
- $\lambda w \lambda t [[^0\text{Bachelor}_{wt} \text{ } ^0\text{Tina}] \rightarrow [^0\text{Man}_{wt} \text{ } ^0\text{Tina}]]$
- $*\text{Cons}_0(A): \{[^0\text{Bachelor}_{wt} \text{ } ^0\text{Tina}]\}$
- $*\text{Cons}_0(C): \{[^0\text{Man}_{wt} \text{ } ^0\text{Tina}]\}$
- $*\text{Cons}_1(A)$ (refined Bachelor):
 $\{[^0\text{Bachelor}_{wt} \text{ } ^0\text{Tina}], [[^0\text{Unmarried}_{wt} \text{ } ^0\text{Tina}] \wedge$
 $[^0\text{Man}_{wt} \text{ } ^0\text{Tina}]], [^0\text{Unmarried}_{wt} \text{ } ^0\text{Tina}], [^0\text{Man}_{wt} \text{ } ^0\text{Tina}]\}$
- $*\text{Cons}_0(C) \subseteq *\text{Cons}_1(A)$

(3^{*}): a TIL analysis of analytically true conditionals

- $\lambda w \lambda t [[[^0Adult_{wt} \ ^0Tim] \wedge [^0Kitten_{wt} \ ^0Tim]] \rightarrow [^0Kitten_{wt} \ ^0Tim]]$
- The $*Cons_0(A)$ of the hyperproposition in the antecedent are as follows:
 $*Cons_0(A): \{ [[^0Adult_{wt} \ ^0Tim] \wedge [^0Kitten_{wt} \ ^0Tim]], [^0Adult_{wt} \ ^0Tim], [^0Kitten_{wt} \ ^0Tim] \}$
- And the $*Cons_0(C)$ of the hyperproposition in the consequent are as follows:
 $*Cons_0(C): \{ [^0Kitten_{wt} \ ^0Tim] \}$

(3*): a TIL analysis of analytically true conditionals

- The consequences $*Cons_0(C)$ of a hyperproposition in the consequent are contained in the consequences $*Cons_1(A)$ of the hyperproposition in the antecedent ($*Cons_0(C) \subseteq *Cons_1(A)$), so the conditional is analytically true, despite the hidden inconsistency in the antecedent.
- If we were to derive further consequences, this inconsistency would come to light (using refinement):
 $*Cons_1(A): \{ [[^0Adult_{wt} ^0Tim] \wedge [^0Kitten_{wt} ^0Tim]], [^0Adult_{wt} ^0Tim], [^0Kitten_{wt} ^0Tim]], [[\neg ^0Adult_{wt} ^0Tim] \wedge [^0Cat_{wt} ^0Tim]], [[\neg ^0Adult_{wt} ^0Tim], [^0Cat_{wt} ^0Tim]] \}$
- The set $*Cons_1(A)$ contains a contradiction: Tim is both adult and is not. An impossible antecedent does not prevent the conditional from being analytically true.

(4): a TIL analysis of analytically true conditionals

- This also holds if the contradiction is explicit, as it is in $[[A \wedge \neg A] \rightarrow A]$
- The $*Cons_0(A)$ of the hyperproposition in the antecedent are as follows:
 $*Cons_0(A): \{[A \wedge \neg A], A, \neg A\}$
- And the $*Cons_0(C)$ of the hyperproposition in the consequent are as follows:
 $*Cons_0(C): \{A\}$
- We can see that the consequences $*Cons_0(C)$ of a hyperproposition in the consequent are contained in the consequences $*Cons_0(A)$ of the hyperproposition in the antecedent, $*Cons_0(C) \subseteq *Cons_0(A)$.

(5): a TIL analysis of analytically true conditionals

- $\lambda w \lambda t [[^0\text{Obligatory}_{wt} \lambda w \lambda t [[^0\text{Cleans}_{wt} ^0\text{Tilman}] \wedge$
 $[^0\text{Cooks}_{wt} ^0\text{Tilman}]]] \rightarrow$
 $[^0\text{Obligatory}_{wt} \lambda w \lambda t [^0\text{Cooks}_{wt} ^0\text{Tilman}]]]$ (wts simplified for
the purpose of the talk)
- $*\text{Cons}_0(A): \{[^0\text{Obligatory}_{wt} \lambda w \lambda t [[^0\text{Cleans}_{wt} ^0\text{Tilman}] \wedge$
 $[^0\text{Cooks}_{wt} ^0\text{Tilman}]]]\}$
- $*\text{Cons}_0(C): \{[^0\text{Obligatory}_{wt} \lambda w \lambda t [^0\text{Cooks}_{wt} ^0\text{Tilman}]]\}$

(5): a TIL analysis of analytically true conditionals

- $*Cons_1(A)$ (deontic reasoning):
 $\{ [{}^0Obligatory_{wt} \lambda w \lambda t [[{}^0Cleans_{wt} {}^0Tilman] \wedge$
 $[{}^0Cooks_{wt} {}^0Tilman]]], [{}^0Obligatory_{wt} \lambda w \lambda t [{}^0Cleans_{wt} {}^0Tilman]],$
 $[{}^0Obligatory_{wt} \lambda w \lambda t [{}^0Cooks_{wt} {}^0Tilman]] \}$
- $*Cons_0(C) \subseteq *Cons_1(A)$

Concluding remarks

- The first steps towards a logic of analytically true conditionals in TIL
- leading idea: content-related relationships matter for the truth of conditionals, and some conditionals are even true just because of these content-related relationships; such conditionals are analytically true
- TIL possesses means for capturing these
- Inspiration taken from Duží et al (2021), which developed a method for reasoning about requisites of a concept (requisites capture essential relationships, such as those between the two properties, such as cats and mammals) even when the concepts in question are necessarily empty

Concluding remarks

- The hyperintensional variant of requisites is called hyperrequisites
- This method is particularly helpful when one aims to capture conceptual relationships between antecedents and consequents of conditionals
- To move from concepts to hyperpropositions, we traded concepts and hyperrequisites for consequences of hyperpropositions
- Future research is needed to explore empirically true conditionals: the framework would have to be extended beyond the content-related relationships (cf. for instance, Gahér (2022) and the need for an additional parameter as well as the principle *ceteris paribus*)

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ĎAKUJEM
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